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Ancient China history-based task to support students' geometrical reasoning and mathematical literacy in learning Pythagoras

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Ancient China history-based task to support students' geometrical reasoning and mathematical literacy in learning Pythagoras

A D Fachrudin¹, R Ekawati², A W Kohar², S Widadah¹, I B Kusumawati¹, R Setianingsih²

¹Mathematics Education Department, STKIP PGRI Sidoarjo, Indonesia
²Faculty of Mathematics and Natural science, Universitas Negeri Surabaya, Indonesia

e-mail: dhany@stkippgri-sidoarjo.ac.id

Abstract. This study aims to identify the potential learning opportunities provided by ancient china's geometric diagram introduced by Liu Hui into student task in learning Pythagorean theorem. We investigate the role of the history-based task, from *Nine Chapter*, in supporting students' mathematics literacy and geometrical reasoning. Design research was chosen as a method to reach the purpose. Our discussion is limited only from a part of one phase, namely pilot experiment, from three main phases of design research. This research was conducted at three different Junior High schools in Indonesia. The instructional activities were designed based on history-based problem-solving. The analysis was focused on students' learning process, and specifically, the results of their visual reasoning and mathematical literacy. The result indicated that the use of figures or diagrams, with written information act as a support for performing algebraic calculations for students who are unfamiliar using symbol or algebraic operation. In this way, by using geometric manipulation, a new figure is arrived at on which the solution can now be read easier. Furthermore, by using history-based context, students got more opportunities to improve their performance in mathematics literacy by formulating various contextual mathematics problem, sourced from history, into mathematical form.

1.　Introduction

History of Mathematics (HoM) provides situations for students gaining a wealthy experience and understanding of the development of mathematical concepts and their connections. Therefore, the integration of HoM in classrooms become an interesting topic for learning designers or researchers to develop an instructional task inspired the HoM. As an example, Fiangga [1] using a historical perspective to design learning activities on the second-degree integral topic. Radford and Guérette [2] developed a set of instructional tasks in learning second-degree equation that inspired by Babylonian Approach. On the other hand, Fachrudin, Putri, Kohar and Widadah [3] also using another perspective of the Babylonian approach to developing a design of learning of quadratic equation. The results of all the studies mentioned, in line with the statement from Man-Keung [4], indicated that the use of HoM in the classroom does not cause students acquire high scores in certain topic overnight, but it encourages learning mathematics to be meaningful, consequently learning process will come easier and encourages. History of Mathematics (HOM) provides conditions for gaining developing rich experience and understanding of the development of mathematical concepts.

　One of the HoM topic recommended by Katz [5] is the ninth chapter (*Gou Gu*) of Jiuzhang suanshu or known as "Nine Chapter" for learning Pythagorean theorem which twenty-four problems concerning the properties of right triangle are presented. Liu Hui, in the last chapter of "Nine Chapter", gave commentary concerning geometric manipulation method in solving the various problem related to right

triangle in Gou Gu. Guevara-Casanova & Burgués-Flamarich [6] encourage to introduce the geometric manipulation in supporting students' visual reasoning rather than calculating with algebraic symbols. On the other hand, Siu [7] argued that the use of Chinese problem solving from Nine Chapters can support student visual reasoning. Therefore, using Lui Hui's diagram procedure and problems presented in Gou Gu student can associate their symbolic algebraic thinking with visual thinking about area of shape to build their understanding related to the Pythagorean Theorem.

The use of diagrams will be a bridge for students in developing their algebraic knowledge from geometric knowledge. The study of Guevara-Casanova & Burgués-Flamarich [6] indicated that the way of introducing algebra with geometrical interpretations may be profitably applied in the classroom. Meanwhile, Radford and Gerette [2] argued that the use of the geometrical approach from the history of mathematics provides a useful context to help the students developing the meaning of mathematical symbols.

One of the mathematical content categories in the Program International Student Assessment (PISA) for measuring students' mathematics literacy is space and shape. Based on the study of students' mathematical literacy skills conducted by the OECD, the average Indonesian student is at level 1 or below out of 6 levels, lag far behind from some Southeast Asian countries (i.e. Malaysia, Thailand, and Singapore). From the 2012 PISA study, where mathematics is the main domain of assessment, in the context of geometry, known as space and shape in PISA, around 70% of Indonesian students are on level 1 and below. This means that their ability was at the stage of recognizing and solving simple problems in a familiar context using pictures or drawings of familiar geometric objects and applying basic spatial skills.

Through various problems and geometric diagrams of ancient China, we tried to design a series of instructional tasks to study the Pythagorean theorem. Concerning the geometrical manipulation of ancient China, we hope students can associate their symbolic algebraic thoughts with visual thinking about geometric shapes to build their understanding related to the Pythagorean Theorem. On the other hand, using various contexts from the history of mathematics, students have the opportunity to develop their mathematical literacy skills. Specifically, the purpose of this study is to describe how a series of instructional tasks based on ancient china history can support Students' Geometrical Reasoning and Mathematical Literacy in Pythagoras Learning.

2.   Ancient China History-based Task

The *Jiuzhang suanshu* or known as Nine Chapter on Mathematical Art is the ancient Chinese that consists of nine different part. In the last chapter, or also known as *Gou Gu* chapter, consists of twenty-four problem solving concerning about right triangle's properties [8]. We will discuss Liu Hui's argument and commentaries of the problem presented in *Gou Gu* which describes the solution to the right triangle problem using diagram and becomes evidence that ancient China recognized the Pythagorean theorem. The geometric interpretation of Liu's argument (from [5]) which is proof of the Pythagorean theorem we present below.
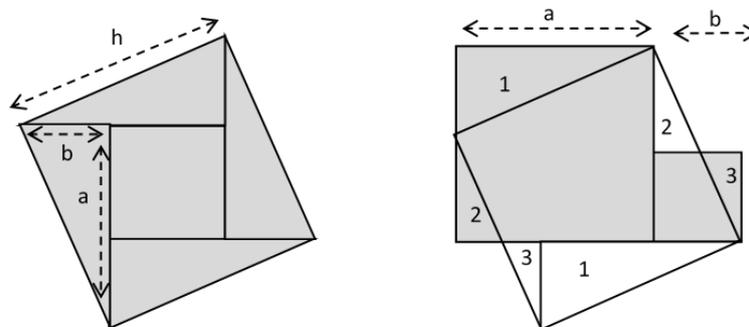

Figure 1. Diagram Representation of Liu's Argument.

Based on figure 1, Liu wants to show that $h^2 = a^2 + b^2$ or $h^2 = (a - b)^2 + 2ab$. By concerning the geometric representation of Liu's argument, we can conclude that mathematician from ancient China has known the basic concepts in the Pythagorean theorem and even one of the simple algebraic identities. In the *Gou Gu* chapter, the sort side adjacent to the right angle known as *Gou*. The longer side adjacent to the right angle is *Gu*. The side opposite the right angle is *Xien*. *Xien* is longer than *Gu*, and *Gu* is longer than *Gou* [5].

The basic of *Gou Gu* Rule: **add the square of *Gou* and *Gu*. The square root of the sum is equal to *Xien*.**
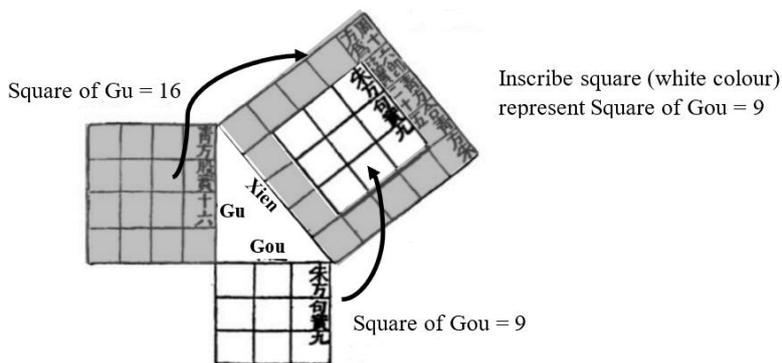

Figure 2. Relation among *Xien*, *Gu,* and *Gou (image source*: [8]).

Base on the geometric interpretation above, it is known that the *Gou Gu* rule has similarity with Pythagorean Theorem. The pairs of right triangles side, 3, 4, and 5 are used as a case of examples to show the proof and have been widely known by many ancient societies.

Additionally, we will show an example of Liu Hui's explanation of the solution to the sixth problem of Gou Gu in Nine Chapter.

in the center of a square pond whose side is 10 *Chi* (feet), grows a reed whose top reaches 1 *Chi* above the water level. if we pull the reed toward the bank, its top is even with the water's surface. what is the depth of the pond and the length of the plant?
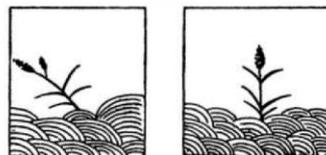
1 Chi = 1 Feet

Figure 3. Problem 6 of *Gou Gu*.

To solve the type of problem using algebraic symbols, indeed students need prior knowledge about product formula of algebraic sum or difference. However, rather than using calculation of algebraic symbols, we may imagine the right triangle devised by Liu Hui as that described by Swetz and Kao [8], Dauben [9], Chemla and Gou [10], Guevara-Casanova & Burgués-Flamarich [6], although Liu Hui did not explain any figures in his commentary.
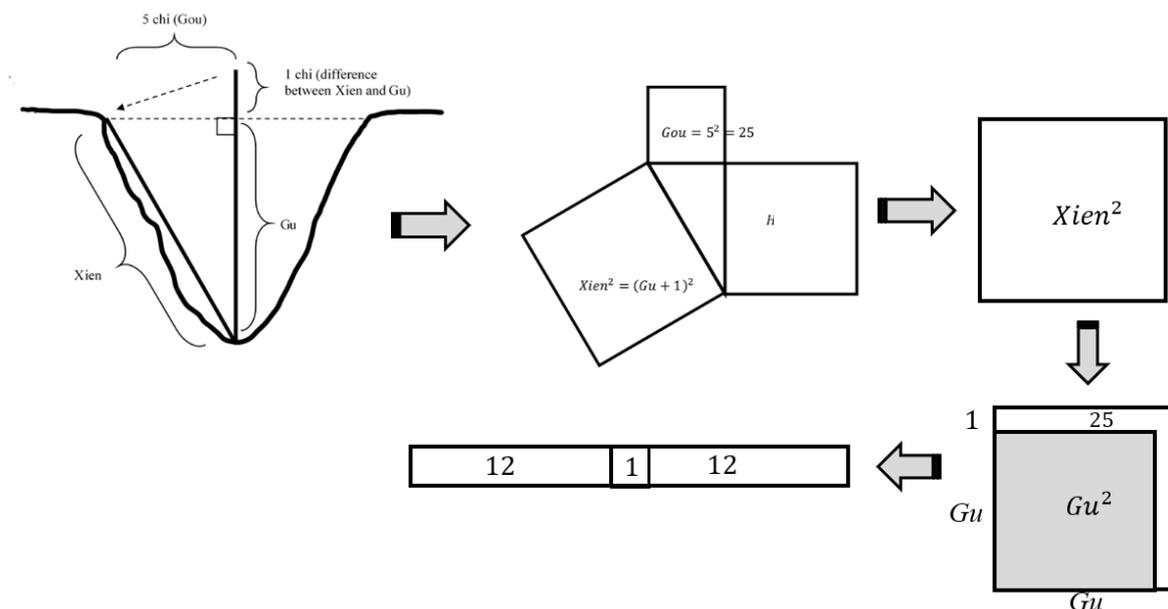
Figure 4. Geometric illustration and method to find *Gou* and *Xien* the problem 6.

We assume that the length of the reed is *Xien* and the depth pond is Gu. Since the sum of the areas of *Gou* square and *Gu* square of the triangle should equal to *Xien* square, by placing the *Gu* in *Xien*, we know that the rest of the area (L shape area or gnomon, see Fig.4) must be 25 chi-square. Then by reshaping the gnomon into a rectangle, with length = 25 ft and width = 1 ft), it will be easily obtained that *Gu* must be 12 ft (see figure 4). Therefore, it will be easy to determine the value of *xien* is 13 Chi. We believe the geometric manipulation based on Liu's argument can support students' geometric reasoning in solving the problems related to the right triangle.

3.   Mathematics Literacy and Problem-Solving from the History of Mathematics

OECD [11] defined mathematics literacy as an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. If we discuss deeply about context, Van Den Heuvel-Panhuizen [12] stated that context can be used as a first step to build mathematical concepts. On the other hand, CORD [13] argued that knowledge will be meaningful to students if the learning process is carried out by involving a situation or context. Problems in the History of Mathematics can be used as contexts because they are included in the category of educational situations. By involving historical problems in learning, it will also provide several benefits, including alternative solutions to problems that may arise in learning can be identified because the obstacles or problems often faced by students are almost the same as people in the past [14,15]. In other words, integrating literacy into mathematics educations can be done by presenting problem-solving from the history of mathematics as a context. Therefore, it will promote a constructivist vision of learning.

4.   Method

The method used in this research was design research. The main aim of this type of research is developing a Local Instruction Theory (LIT) [15] by testing and comparing the Hypothetical Learning Trajectory (HLT) that has been developed with the actual learning through teaching experiments. However, our discussion in this paper is restricted only from a part of the teaching experiment, namely pilot experiment, from three main phases of design research.

This research was conducted at three different Junior High schools in Indonesia and involving 17 students. The data collected was field note, students' work, and video recordings. After testing the HLT, we conducted retrospective analysis to compare the actual learning in every task with the HLT. We use the results of the analysis to revise the HLT. In this paper, we not only conducted qualitative analysis

but also quantitative impression of how well the conjecture and actual learning matched based on Bakker & Van Eerde [16]. Moreover, we did qualitative analysis to describe how the instructional tasks based on ancient china history can support Students' Geometrical Reasoning and Mathematical Literacy in Pythagoras Learning.

5.   **Result and Discussion**
In this section, we will explain briefly the HLT and student thinking conjectures that we compiled to support students' understanding of the Pythagorean theorem. Furthermore, we will discuss deeply about how a series of tasks we developed can support students' visual reasoning and mathematics literacy skills.

5.1.   Learning Trajectory
The following is the explanation of the series of the instructional task we developed and analysis of the comparison between HLT and Actual Learning Trajectories (ALT).
a.   Activity I
In the first activity, we ask the students to prove the *Gou Gu* rule using the concept of area. The purpose of this activity is to emphasize that there are various models of geometric interpretation in proving that the area of *Xien* is equal to the sum of the area of *Gou* and *Gu*. The conjectures of student thinking in this activity are following.
C.1.1. Students prove that the sum of the area of Gou and Gu is equal to the area of Xien.
C.1.3. Students explain that the Gou Gu rule applies to right triangles only by showing that the rule does not apply to the acute triangle and obtuse triangle.
b.   Activity II
In this activity, students are asked to solve a contextual problem (the vine problem) of right triangle using *Gou Gu* rule. This activity aims to help students achieving a better understanding in mathematics literacy by solving contextual problems. the conjectures of student thinking were:
C.2.1. Students solve problems no.1 in activity II by using the *Gou Gu* rule.
C.2.1. Students solve history-based contextual problems (no.2) in activity II by using the Gou Gu rule.
c.   Activity III
In the third part, students were asked to solve several contextual right triangle problems from *Gou Gu*. The case of the tasks is more difficult than the previous ones since the information given involve the difference between two sides of the right triangle. Guevara-Casanova & Burgués-Flamarich (2018) argued that the difficulty can be overcome by introducing geometric approach to support the students to argue visually and follow the procedure for finding the solution to the problem by manipulating the area of shapes.
C.3.1. Students performing geometric manipulation in solving the history-based contextual problems (no.1) in activity III.
C.3.2. Students performing geometric manipulation in solving the history-based contextual problems (no.2) in activity III.
d.   Activity IV
The last activity aims to introduce a variety of triple Pythagoras by combining three squares to form a right triangle.
C.4. Students are able to compose a square pair so that they get a Pythagorean triple number and find more than 3 pairs of triple Pythagoras (which are obtained not from multiplying that are known).

Based on the pilot experiment, the following is the quantitative data analysis of comparison between HLT and ALT.

Table 1. ALT result compared with HLT conjectures for the tasks involving a particular type of reasoning

Note: an x means how well the conjecture matched to the observed learning (− refers to confirmation for up to 1/3 of the total students, and + to at least 2/3 of the total students)

Based on the table above, we concluded that in general the actual learning trajectory has been running in accordance with the conjectures in HLT, except for the tasks in activity IV. This is because, during the pilot experiment, we made a mistake in making a square model (as a prop). The difference of the length between the model that we designed in the word file and the printout result caused the triple Pythagoras compiled by students do not match with what we have planned. Therefore, we revise the HLT related to the media we use in activity 4.

5.2.   Solving History-based Problems of Right Triangle with Diagram: The Example of Students' Work

We will explain how students solve problem-solving using geometric manipulation or diagram of two problems taken from the test in activity 3 (see reed in the pond problem). During the test, the students worked individually and some of them in pair. The first task in this activity is taken from problem 6 of Gou Gu. The following are examples of students' answers in solving these problems.
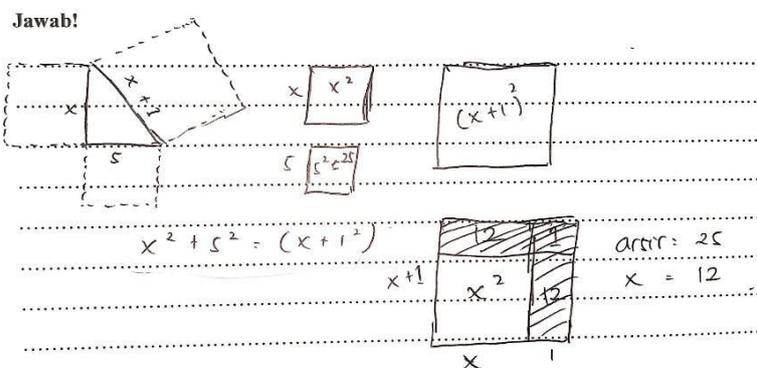

Figure 5. Student's visual reasoning in solving the first problem in activity III.

Based on the picture above, the first step taken by students was formulating the contextual problems into a geometric problem (mathematical form) of a right triangle. The right triangle with one side is known (5 chi) and two sides are unknown. Student determines the deep of the pond as x feet, and the height of the reed as $x+1$ chi. The problem is determining the x such that the deep of the pond and the height of the reed can be determined. When referring to the procedures of solving problem with diagrams of Guevara-Casanova & Burgués-Flamarich [6] we found the similarities. The picture (figure.5 and 6) show that in solving the problem in early-stage student formulate the contextual problem into mathematical form, the right triangle problem. because the stage requires the ability to transform contextual forms into mathematical problems, we call it a *translation* procedure. Secondly, students use the relationship between the area of "square *gou*, *gu*, and *xien*, and perform a geometric manipulation. The step resulted that the gnomon form (L shape area) has the same area as the *gou* square. We call that step as a *transformation* procedure. In the end, students analyzed the relation between the area of the

gnomon and the side is given, they found that $x = \frac{55}{2} \times \frac{1}{3}$ or they wrote $x = 9,16$ (actually the most appropriate answer is $x \approx 9,167$) (see figure 5).
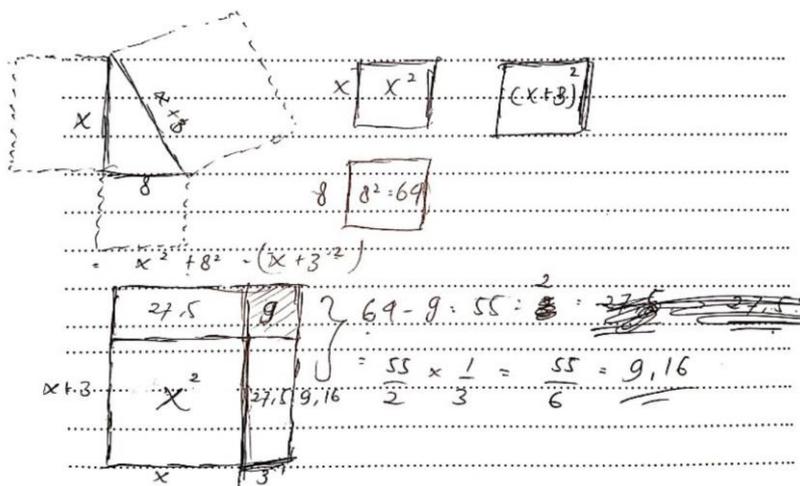

Figure 6. Student's visual reasoning in solving the second problem in activity III.

5.3. Mathematics Literacy and problem-solving in history-based task

The following are examples of historical problems that we profile based on content, context and processes in mathematics literacy.

Problem 1.
>   A tree of height 20 feet has a circumference of 3 feet. There is a vine which winds seven
>   times around the tree and reaches the top. What is the length of the vine?

Item Profile:

Description	: Using Pythagorean theorem in solving problem related to a context-based task
Context	: Scientific
Content	: Space and Shape
Process	: Employ
Level of context use	: first of context use

This task is categorized to have a context of science since it is related to a discipline of biological science, which show that a vine plant can grow up with watery environment. Furthermore, it is included in the content of space and shape because it demands the solver to use their conceptual understanding of the measurement of triangle. Although this task can be formally solved using the Pythagorean theorem, a solver could also use an informal way to solve the problem. In this task, a solver needs more effort in using mathematical computation in obtaining mathematical results, thus it is categorized in the group of 'employ' task. Regarding the level of context use, this task is indicated to have the first level of context use because the context of satiation describing the geometrical information about the bamboo is needed for solving the problem and judging the answer.

Problem 2.
>   In the centre of a square pond whose side is 10 *Chi* (feet), grows a reed whose top
>   reaches 1 *Chi* above the water level. if we pull the reed toward the bank, its top is even
>   with the water's surface. What are the depth of the pond and the length of the plant?

Item Profile:

Description   : Using Pythagorean theorem in solving problem related to a context-based task
Context   : Scientific
Content   : Space and Shape
Process   : Employ
Level of context use   : first of context use

This task is categorized to have a context of science since it is related to a discipline of biological science, which show that a bamboo plant can grow up with watery environment. Furthermore, it is included in the content of space and shape because it demands the solver to use their conceptual understanding of the measurement of triangle. Although this task can be formally solved using the Pythagorean theorem, a solver could also use an informal way to solve the problem. In this task, a solver needs more effort in using mathematical computation in obtaining mathematical results, thus it is categorized in the group of 'employ' task. Regarding the level of context use, this task is indicated to have the first level of context use because the context of satiation describing the geometrical information about the bamboo is needed for solving the problem and judging the answer.

In conclusion, the students 'ability to solve contextual problems from the history of mathematics is in line with the students' mathematics literacy. In this case, it is limited to the specific context, content and process, based on 1 and 2 level of context.

6.   **Conclusion and Implications**

From the result regarding the implementation in pilot experiment of instructional task we developed, we conclude that problem-solving from the history of mathematics can support students' understanding of the concept of Pythagoras. The visual argument by using concept of area in solving the right triangle problem help student improve their visual reasoning through three main procedures namely *translation, transformation,* and *diagrammatic reasoning*. On the other hand, the history-based problem as context students get a chance to learn Pythagorean theorem meaningfully. This is aligning with Freudenthal [17] statements that learning will occur when meaningful for students. On the other hand, HoM plays important role in improving students' mathematical literacy skills through contextual problems in HoM that are aligned with real-world challenges in PISA. Additionally, problem-solving from HoM also support students' mathematics literacy through process categories (i.e. formulate, employ, and interpret mathematics in a variety of contexts) of mathematics literacy.

In conclusion, this study revealed that Ancient China Pythagoras has a plentiful didactical source to improve the quality of mathematics pedagogy, especially for designing the instructional task for learning Pythagorean theorem. Hence, we recommend teachers to integrate the history of mathematics in the classroom and researcher or learning designer to conduct other similar research to enrich teachers' references in using history of mathematics in classroom.

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